

Transmissivity  $\tau$

Absorbed  $\alpha=1-\tau$

Internal heat source of surface =  $238\text{W/m}^2$

$\varepsilon\sigma$

Incoming radiation to shell:

$\sigma T_s^4$

Absorbed by shell is:

$\alpha\sigma T_s^4$

Balance of shell is:

$$\sigma T_L^4 = \alpha\sigma T_s^4 \quad \text{or} \quad T_L^4 = \alpha T_s^4$$

Emission of shell is:

$$\varepsilon_L = \sigma T_L^4 \quad \text{or} \quad \varepsilon_L = \sigma\alpha T_s^4$$

$$\varepsilon_L = \alpha\varepsilon_s$$

Incoming radiation to surface:

$$238 + \text{Incoming from shell} \quad \text{or} \quad 238 + \varepsilon_L/2$$

Absorbed by surface:

$$238 + \varepsilon_L/2$$

Balance of surface:

$$\sigma T_s^4 = 238 + \varepsilon_L/2 \quad \text{substitute for } \varepsilon_L$$

$$\sigma T_s^4 = 238 + \sigma\alpha T_s^4/2 \quad \text{solve for } T_s$$

$$238 = \sigma T_s^4 - \sigma\alpha T_s^4/2 \quad \text{combine left side}$$

$$238 = (2\sigma T_s^4 - \sigma\alpha T_s^4)/2 \quad \text{factor out } T_s^4 \text{ and } \sigma$$

$$476 = T_s^4\sigma(2-\alpha) \quad \text{isolate } T_s^4$$

$$476/\sigma(2-\alpha) = T_s^4 \quad \text{fourth root and sub } \alpha$$

$$T_s = (476/\sigma(2-(1-\tau)))^{.25}$$

Now let's solve for the total emission,  $\varepsilon_T$ , at the top:

$$\varepsilon_T = \tau\varepsilon_s + \varepsilon_L/2 \quad \sigma T_s^4 = 238 + \varepsilon_L/2$$

$$\varepsilon_T = \tau\sigma T_s^4 + \sigma T_s^4 - 238 \quad \text{factor out } \sigma T_s^4$$

$$\varepsilon_T = \sigma T_s^4(\tau + 1) - 238 \quad T_s = \sqrt[4]{\left(\frac{476}{\sigma(1+\tau)}\right)}$$

$$\varepsilon_T = \sigma((476/\sigma(1+\tau))^{.25})^4(\tau + 1) - 238$$

$$\varepsilon_T = \sigma(476/\sigma(1+\tau))(\tau + 1) - 238$$

$$\varepsilon_T = \sigma(476/\sigma) - 238$$

$$\varepsilon_T = 476 - 238$$

$$\varepsilon_T = 238$$

Quantity of emission is independent of transmissivity. Once the system reaches steady state the emission out the top will be  $238\text{W/m}^2$ .

$$T_s = \sqrt[4]{\left(\frac{476}{\sigma(1+\tau)}\right)}$$