Irradiance of the Sun, J^*_S, according to Boltzman black body radiation equation:

$$J_S^* = T_s^4 \sigma$$

where T_S = effective temperature of the Sun (5778 K) and σ = Stefan-Boltzman constant (6.669 x 10^{-8} $Wm^{-2}K^{-4}$ and where J^*_S is in Wm^{-2}.

Power emitted by the surface of the sun, P_S:

$$P_S = J_S^* \cdot A_S = T_s^4 \sigma \cdot 4\pi r_S^2$$

where r_S = radius of the Sun (6.960x10^8 m) and P_S is in W.

Irradiance at the orbit of the Earth, J^*_{OR}:

$$J_{OR}^* = \frac{P_S}{A_{OR}} = \frac{T_s^4 \sigma \cdot 4\pi r_S^2}{4\pi r_{OR}^2} = T_s^4 \sigma \cdot \left(\frac{r_S^2}{r_{OR}^2} \right)$$

Where r_{OR} = mean orbital radius of the Earth around the Sun (1.496 x 10^{11} m). $J^*_{OR} \cong$ 1368 Wm^{-2}.

Absorption of irradiant energy by the Earth, $P_{E(ab)}$:

$$P_{E(ab)} = \varepsilon(\lambda) \cdot J_{OR}^* \cdot A_{E(ab)} = \varepsilon(\lambda) \cdot T_s^4 \sigma \cdot \left(\frac{r_S^2}{r_{OR}^2} \right) \cdot \pi r_E^2$$

Where $\varepsilon(\lambda)$ = absorptivity of the Earth and r_E = radius of the earth.

For a black body, the power emitted by Earth, $P_{E(em)}$:

$$P_{E(em)} = \varepsilon'(\lambda) \cdot J_E^* \cdot A_{E(em)} = \varepsilon'(\lambda) \cdot T_E^4 \sigma \cdot 4\pi r_E^2$$

Where r_E = radius of the Earth, T_E = mean temperature of the Earth, and $\varepsilon'(\lambda)$ = emissivity of the Earth.

In thermal equilibrium, $P_{E(ab)} = P_{E(em)}$, where $P_{E(em)}$ = power emitted by the Earth.

$$P_{E(em)} = P_{E(ab)} = \varepsilon(\lambda) \cdot T_s^4 \sigma \cdot \left(\frac{r_S^2}{r_{OR}^2} \right) \cdot \pi r_E^2 = \varepsilon'(\lambda) \cdot T_E^4 \sigma \cdot 4\pi r_E^2$$

Combining and canceling terms, you end up with the following equation for the temperature of the Earth, T_E:

$$T_E = T_S \sqrt{\frac{r_S}{2r_{OR}}} \left(\frac{\varepsilon(\lambda)}{\varepsilon'(\lambda)} \right)^{\frac{1}{4}}$$

When $\varepsilon(\lambda) = \varepsilon'(\lambda)$, TE ≅ 278.7 K (5.5° C). Still too cold (the average temperature of the Earth is actually about 287.7 K, or 14.5° C), but given the accuracy of the equations, effectively equal to what you generated, Bob. Even though you arbitrarily, without explanation, decided that the effective temperature of the Sun was more like 6000 K in order to make your point.

The problem, though, is that $\varepsilon(\lambda) \neq \varepsilon'(\lambda)$. According to Kirchoff's relationship, absorptivity equals emissivity at a given wavelength. The problem is that the effective absorptivity of the Earth (the absolute absorptivity, adjusted for the material properties of the Earth) is not equal to the effective emmisivity of the Earth. In terms of equations,

$$\varepsilon(\lambda) = \alpha(\lambda) \cdot E(\lambda)$$

and

$$\varepsilon'(\lambda) = \beta(\lambda) \cdot E(\lambda)$$

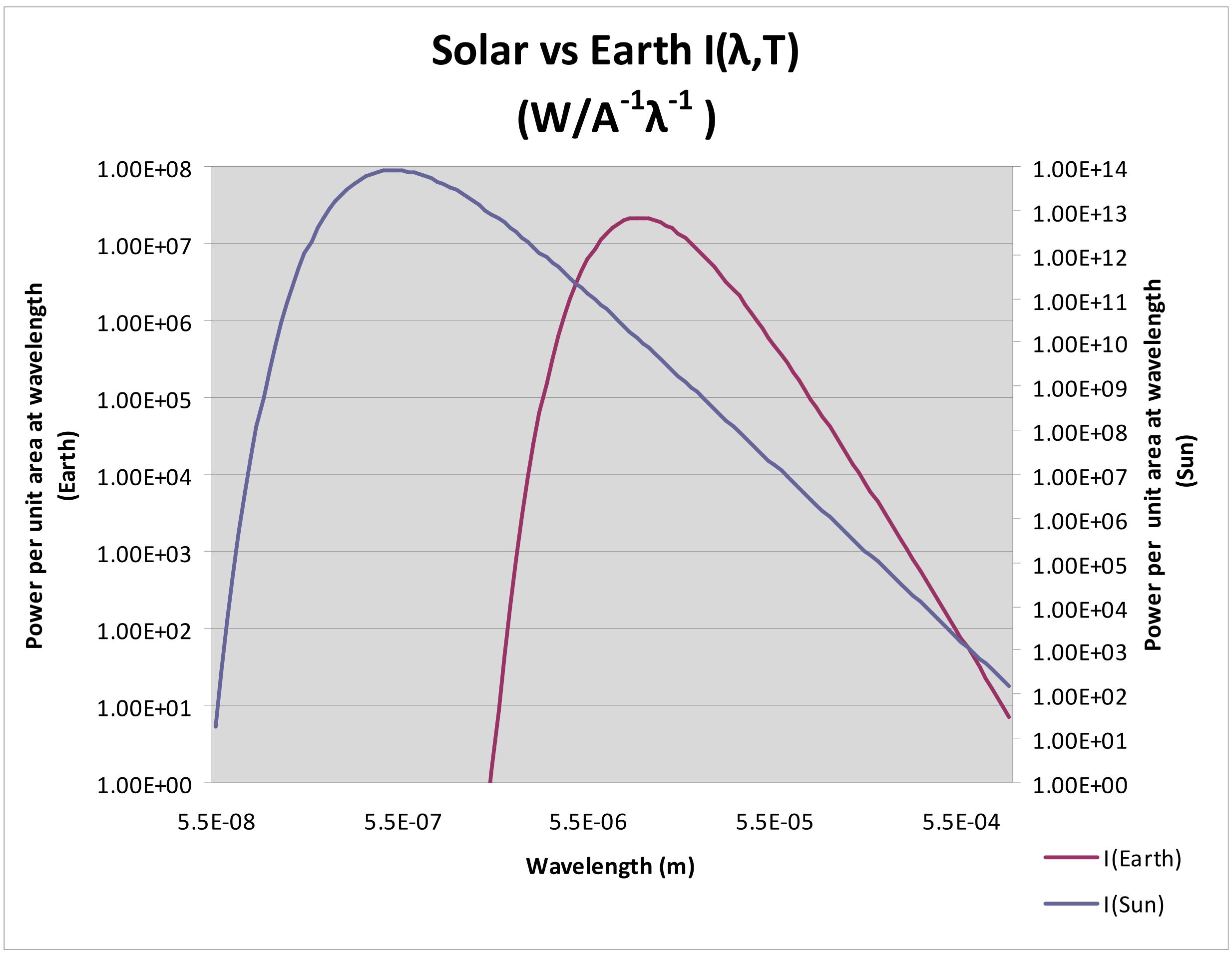
Here's why.

Plank's Law defines how the power spectral density of the emissions of the Sun with respect to λ differ from the power spectral density of emissions from the Earth:

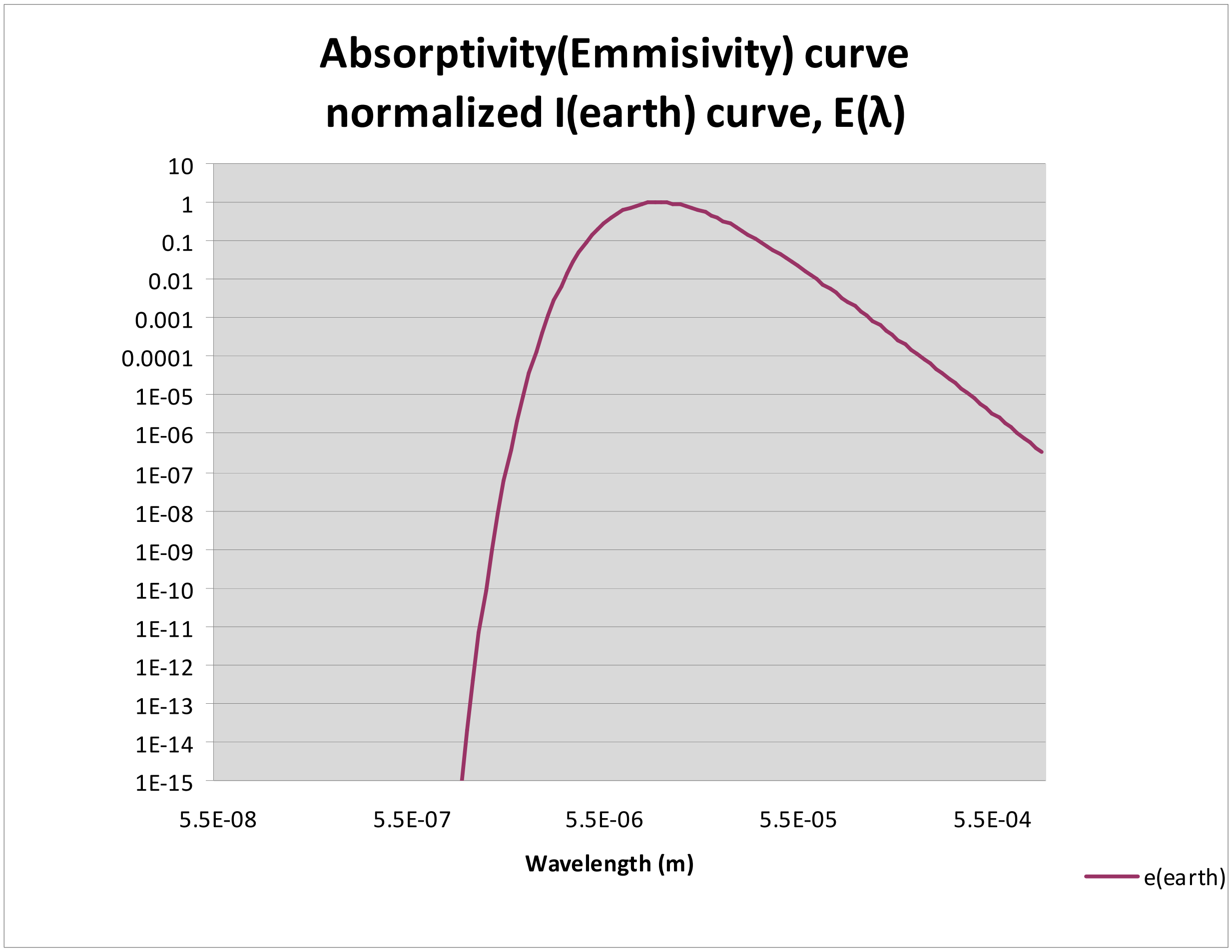
$$I(\lambda,T)=\frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda kT}-1\right)}$$

Where h = Plank's Constant (6.626 x 10^{-34} J*s), c = speed of light (2.998 x 10^8 m/s), and k = Boltzman Constant (1.381 x 10^{-23} J/K), and T is either the temperature of the Sun or the Earth.

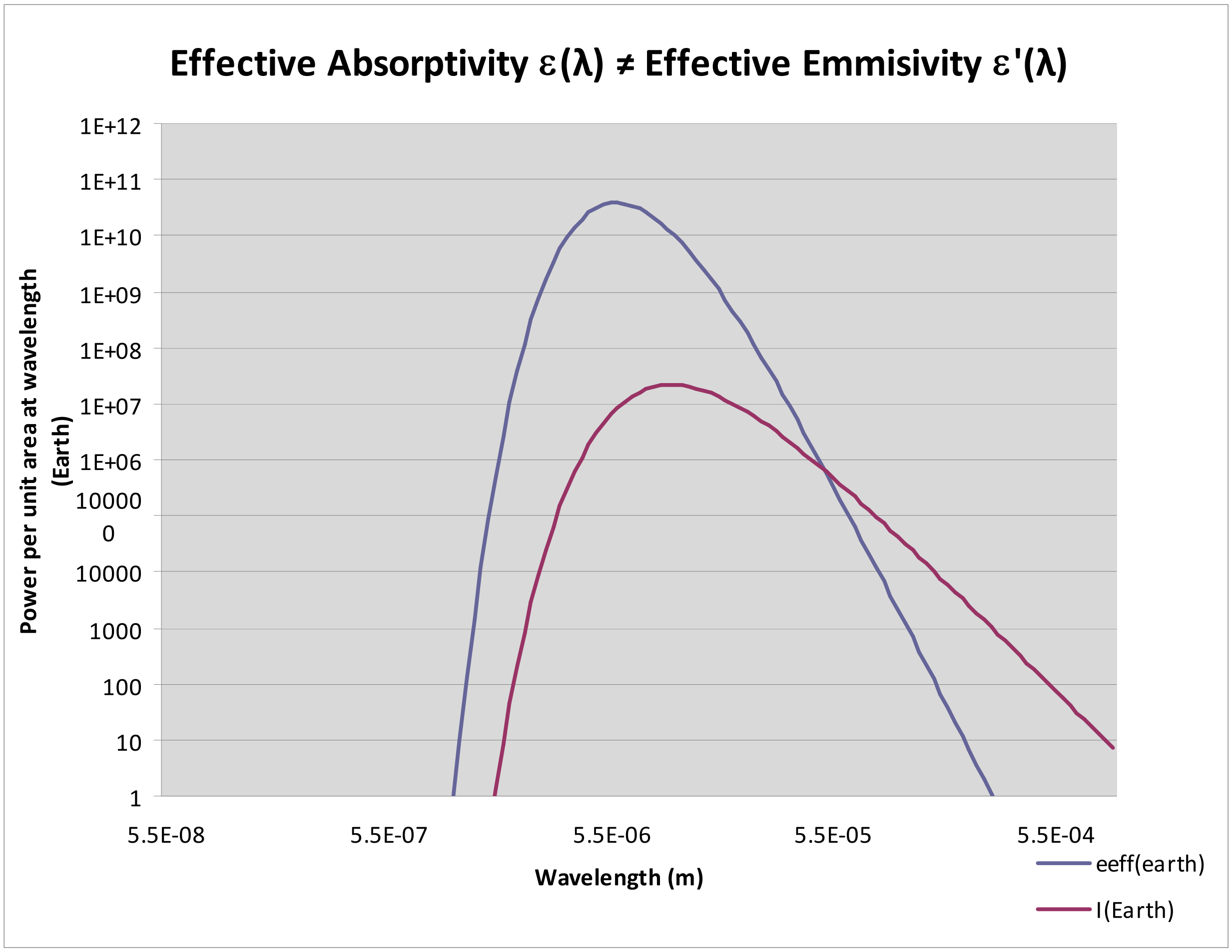
Not only is the Sun's irradiance much higher in magnitude, but it also emits it's peak energy at a much smaller wavelength: approximately 0.5 microns vs. just over 10 microns for the Earth's peak, as illustrated in the first graph below:



Clearly, the two are different, but we can use the power spectral density of the emissions of the Earth as a rough, first order estimate of the emissivity of the Earth (and, according to Kirchoff, of the absorptivity as well). Normalizing the emissivity, we get the following graph:



In order to determine the effective absorptivity, however, we need to multiply the normalized emissivity by the input spectral density of solar radiation on the Earth. When you do that, you get the following graph:



This graph shows that, in fact, effective absorptivity does not equal effective emissivity due to differences in the spectrums of absorption and emission. Therefore, your equations are wrong, as are the conclusions based off them. Furthermore, effective emissivity and effective absorptivity are actually even more different than I just proved due to the actual composition of the atmosphere and the surface of the Earth.